

Resource Multiplying and Dividing Fractions

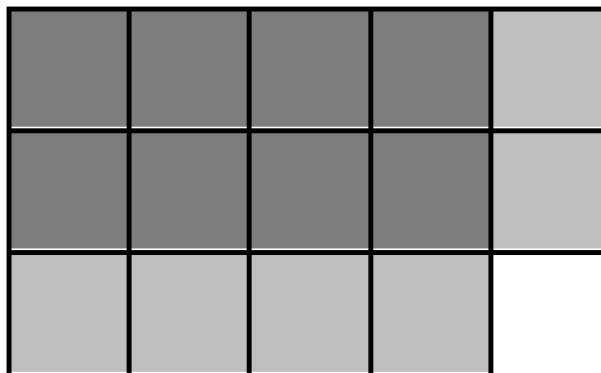
Quantitative reasoning skill: Multiplying fractions

Another way to think about fractions is in terms of area. Look at the rectangle below. The fraction $\frac{2}{3}$ can be represented by the dark gray area found by dividing the rectangle into thirds horizontally and shading 2 of the sections as shown.



You can also think about multiplying fractions in terms of area of a rectangle.

Shade $\frac{2}{3}$ as indicated above. Now represent $\frac{4}{5}$ by dividing the rectangle vertically into fifths and shading 4 of the 5 sections.



The product $\frac{2}{3} \cdot \frac{4}{5}$ can be represented by the region that was shaded twice.

Notice that the rectangle is now divided into 15 sections ($15 = 3 \times 5$), and the number of those regions that are dark gray is 8 ($8 = 2 \times 4$).

So, 8 out of 15 pieces are dark gray, or $\frac{8}{15}$.

This prompts a rule for multiplying fractions: multiply the numerators ($2 \cdot 4$) and multiply the denominators ($3 \cdot 5$), then simplify if possible.

Therefore, $\frac{2}{3} \cdot \frac{4}{5} = \frac{8}{15}$.

Quantitative reasoning skill: Simplifying before multiplying fractions

The fact that common factors in the denominator and numerator of a number can be divided to make 1 can be used to make multiplying fractions easier. Consider the following multiplication problem.

$$\frac{2}{3} \cdot \frac{7}{8} \rightarrow \frac{14}{24} \rightarrow \frac{7 \cdot 2}{12 \cdot 2} \rightarrow \frac{7}{12}$$

If you see that there is a common factor of 2 in the numerator and denominator before multiplying, you can divide the common factors first. This makes the multiplication easier because you have smaller numbers to work with, and the simplification is complete.

$$\frac{2}{3} \cdot \frac{7}{8} \rightarrow \frac{2}{3} \cdot \frac{7}{4 \cdot 2} \rightarrow \frac{2}{3} \cdot \frac{7}{4 \cdot 2} \rightarrow \frac{1 \cdot 7}{3 \cdot 4} \rightarrow \frac{7}{12}$$

This is an important concept when working with ratios with units. You can learn more about this in the Resource **Dimensional Analysis**.

For practice on multiplying fractions, you can view this free video on the Khan Academy website:

- https://www.khanacademy.org/math/arithmetric/fractions/multiplying_and_dividing_frac/v/multiplying-fractions

Additional examples can be found by running an internet search for “multiplying fractions videos” or “multiplying fractions videos.”

Quantitative reasoning skill: Multiplying whole numbers and fractions

If you need some review on multiplying whole numbers and fractions, you can view free videos at the following Khan Academy links:

- <https://www.khanacademy.org/math/cc-fifth-grade-math/5th-multiply-fractions/imp-multiplying-fractions-and-whole-numbers/v/concept-fraction-whole-number-product>
- <https://www.khanacademy.org/math/arithmetric/fraction-arithmetric/arith-review-multiply-fractions/v/multiplying-fractions?playlist=%20Developmental%20Math>

Additional examples can be found by searching “multiply whole numbers and fractions.”

Quantitative reasoning skill: Dividing fractions

Many people struggle with dividing fractions because it is difficult to visualize. A full explanation of the mathematics behind dividing fractions is beyond what the authors can do in these materials. Instead, the authors are providing you with a context that might help you remember how to divide fractions.

Suppose you have \$48 to spend on going to the movies during a month. How many tickets can you buy in a month? A movie ticket costs \$8. One way to think about this is that you want to know how many groups of \$8 there are in \$48, or $48 \div 8$.

In the same way, suppose you had \$10 to spend on downloading songs for a $\frac{1}{2}$ dollar. (For the sake of the mathematics, you are going to express “a half of a dollar” as a fraction instead of as a decimal.) This means you want to know how many $\frac{1}{2}$ dollars there are in \$10. Your common sense probably tells you that the answer is 20 because every 1 dollar has two halves. So, you multiplied 10×2 . Look at this written as a calculation:

$$10 \div \frac{1}{2} \text{ is the same as } 10 \cdot \frac{2}{1}$$

$$\frac{1}{2} \text{ and } \frac{2}{1} \text{ are called reciprocals}$$

So, you say that division is the same as multiplying by the reciprocal. Here are more examples:

$$4 \div \frac{1}{2} \rightarrow 4 \cdot \frac{2}{1} \rightarrow 8$$

$$12 \div \frac{2}{3} \rightarrow 12 \cdot \frac{3}{2} \rightarrow \frac{36}{2} \rightarrow 18$$

$$\frac{4}{5} \div 2 \rightarrow \frac{4}{5} \cdot \frac{1}{2} \rightarrow \frac{4}{10} \rightarrow \frac{2}{5}$$

For practice on dividing fractions and mixed numbers, you can view this free video on the Khan Academy website:

- <https://www.khanacademy.org/math/cc-seventh-grade-math/cc-7th-negative-numbers-multiply-and-divide/cc-7th-mult-div-neg-fractions/v/dividing-mixed-numbers?playlist=Developmental%20Math>

