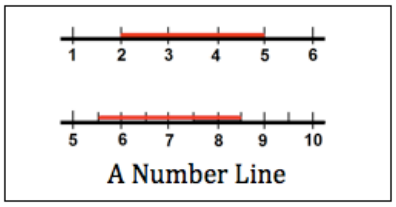
Resource **Length, Area, and Volume**

Length

Length is one-dimensional. An example would be the length of an extension cord that you need to plug in an electronic device. Examples of units of measure for length are inches, feet, yards, or miles (or in the metric system, centimeters, meters, or kilometers).



A Number Line

A **number line** can be used to model lengths.

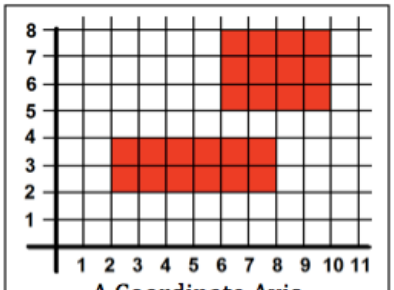
The shaded segment on each number line shown above is 3 units long. If the scale is in inches, each line segment is 3 inches long. If the scale is in feet, each line segment is 3 feet long.

Area

Area is two-dimensional and is measured in square units. The total number of one-foot square tiles needed to cover the floor of a room is an example of area measured in square feet, and can be modeled with a rectangle. Recall the formula for the area of a rectangle:

$$A = L \times W$$

The area of a rectangle is the product of the length and the width, which is a shortcut for counting the number of square units needed to cover the rectangle.



A Coordinate Axis

Each of the two shaded areas on the coordinate axis has an area of 12 square units. If the horizontal and vertical scales are in inches, each area is 12 square inches. If the scales are in feet, each area is 12 square feet. Notice that the regions measured do not have to be squares, yet the area is measured in square units.

If the units are in inches, the area of the top rectangle is:

$A = (3 \text{ inches}) \times (4 \text{ inches})$

$= (3 \times 4) \times (\text{inches} \times \text{inches})$

$= 12 \text{ inches} \times \text{inches}$

$= 12 \text{ square inches}$

If the units are in feet, the area of the bottom rectangle is:

$A = (2 \text{ feet}) \times (6 \text{ feet})$

$= (2 \times 6) \times (\text{feet} \times \text{feet})$

$= 12 \text{ feet} \times \text{feet}$

$= 12 \text{ square feet}$

For more about labeling units, see the note on the next page.

Note 1:

It is common to abbreviate the units of measure using exponents. If the area is

$A = 12$ feet $\times$ feet $= 12$ square feet, we often write $A = 12$ ft^2.

Notice the connection to algebra here!

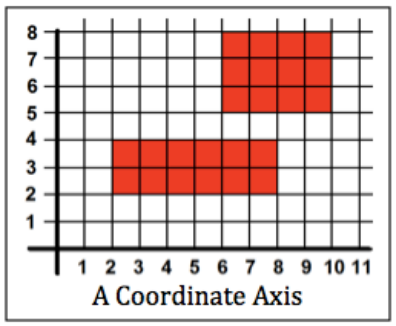
Multiplying $(3$ feet$)$ by $(4$ feet$)$ is similar to multiplying $(3x) \times (4x)$. You multiply the numbers in front of the variables (coefficients), and then multiply the variables:

$$(3x) \times (4x) =$$

$$(3 \times 4)(x \times x) =$$

$$12x^2$$

Note 2:



A Coordinate Axis

Suppose each shaded region represents the dimensions of a room. If you want to know how many floor tiles to buy, area is the correct concept.

But what if you want to trim the edges of the room with baseboards? (If you aren't sure about this term, search the internet for "baseboard images.")

The length of the distance around a shape is called the perimeter. To calculate this length, simply add the total number of units as if traveling around the edge. For example, if the units are feet, then the length of the line around the bottom rectangle is

$$P = 2 \text{ feet} + 6 \text{ feet} + 2 \text{ feet} + 6 \text{ feet}$$

$$= 16 \text{ feet}$$

The arithmetic operation for length is addition, and the unit of measure is feet (you are adding up a lot of feet, so the final result is feet). By comparison, the arithmetic operation to compute area is multiplication, and the unit of measure is square feet (you are determining the number of square tiles). Again, this connects to algebra. To add algebraic terms, you must have like terms, meaning terms with the same variables:

$$2x + 6x + 2x + 6x =$$

$$16x$$

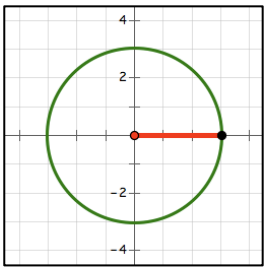
You cannot add $2x + 3y$, since these are not like terms.

Not every shape you need to find the length or the area of will be a square or rectangle. Think about a circular rug in the living room, or a gazebo in the shape of an octagon.

Circle length and area

The distance around a circle is called the circumference, which can be determined using the formula:

$$C = 2\pi r$$



The area of a circle is given by the formula:

$$A = \pi r^2$$

- π is a constant that is approximately 3.14159
  (You probably learned 3.14, but carrying additional decimal places reduces the amount of rounding error.)
- r is the radius of the circle, which varies depending on the size of the circle. It is the distance from the center to any point on the circle.
- C is the circumference of the circle, which varies depending on the radius.
- A is the area of the circle, which varies depending on the radius.

In this example, the radius is 3 units. Let's say those units represent inches.

The circumference is:

$C = 2\pi r$

$C = 2\pi(3 \text{ inches})$

$= 2 \cdot 3 \cdot \pi$ inches

$= 6\pi$ inches (exactly)

$\approx 6 \cdot 3.14159$ inches

≈ 18.85 inches

Look at the length of 1 inch on the radius. Does 19 inches seem like a reasonable estimate for the distance around the circle?

The area is:

$A = \pi r^2$

$A = \pi(3 \text{ inches})^2$

$= \pi(3 \text{ inches}) \times (3 \text{ inches})$

$= \pi \cdot 3 \cdot 3(\text{inches})(\text{inches})$

$= 9\pi$ square inches (exactly)

$\approx 9 \cdot 3.14159$ square inches

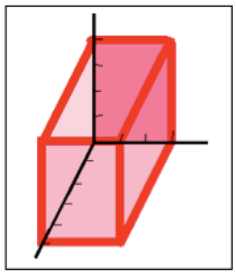
≈ 28.27 in^2

Look at the grid. Is 28 in^2 a reasonable estimate of the area of the circle

Volume

Volume is three-dimensional and is measured in cubic units. The formula to calculate the volume of a box is:

$$V = L \times W \times H$$



If the graph at the right is in inches, then this shape is

5 inches long, 3 inches wide, and 4 inches high.

$V = (5 \text{ inches}) \times (3 \text{ inches}) \times (4 \text{ inches})$

$= 60$ inches^3 or 60 cubic inches

If you need some review on calculating percentage rates, you can view free videos at the following links.

- Area: Square Units and Rectangles
  http://www.youtube.com/watch?v=epeFZ6v7u_U
- Area: Parallelograms and Triangles
  http://www.youtube.com/watch?v=vQC10PPmuoA&feature=related
- Area of Composite Shapes
  http://www.youtube.com/watch?v=by9-_95Zn08&feature=related
- Geometric Solids: Lesson Hook
  http://www.youtube.com/watch?v=RGlU_SqgjEg
- GMAT Prep: Math, Geometry, Rectangular Solids by Knewton
  https://www.youtube.com/watch?v=SJycga8V02M&feature=related