Resource **Four Representations of Relationships**

Expressing mathematical relationships

Mathematical relationships can be represented in four ways: models (algebraic equations), tables, graphs, and verbal descriptions. If you are struggling with a problem, approaching it with a different representation may help you to make sense of the work.

Model or equation

Suppose you wanted to know the mathematical equation that represents the relationship between the price of gas and the cost of driving a car. An equation is useful because it can be used to calculate cost values. In addition to modeling situations, equations are also useful for communicating complex relationships. In writing equations, it is always important to define what the variables represent, including units. For example, in the modeling scenario of price of gas and cost of driving a car above, we can define the variables as shown below. Note that each definition includes what the variable represents, such as cost of driving the car, and the units in which this quantity is measured, such as dollars/mile.

C = Cost of driving the car in dollars/mile

g = Price of gas (dollars/gal)

These variables can then be used in the mathematical equation that models the scenario, $C = \frac{g}{22} + 0.146$. (Assuming the car averages 22 miles per gallon on the highway and the car's general maintenance expenses come out to $\$0.146$ per mile.)

Verbal description

A verbal description explains the relationship in words, which can help you make sense of what the relationship means in the context.

For the equation $C = \frac{g}{22} + 0.146$, the fraction $\frac{g}{22}$ represents the per-mile cost of gas, which depends on the price of gas $\left(\frac{\text{dollars}}{1 \text{ gallon}} \cdot \frac{1 \text{ gallon}}{22 \text{ miles}} = \frac{\text{dollars}}{\text{mile}}\right)$ and 0.146 represents the per-mile costs associated with oil changes, tire wear, etc.

So, the equation $C = \frac{g}{22} + 0.146$ represents the total per-mile cost of driving the car. Verbally, the owner might say, "I need to find the per-mile cost of my car so that I can compare it to the cost of a car rental. Dividing the gas price by 22 miles per gallon will give me the per-mile cost of the gas and then I need to add in the per-mile maintenance costs to get the total."

Continued on the next page.

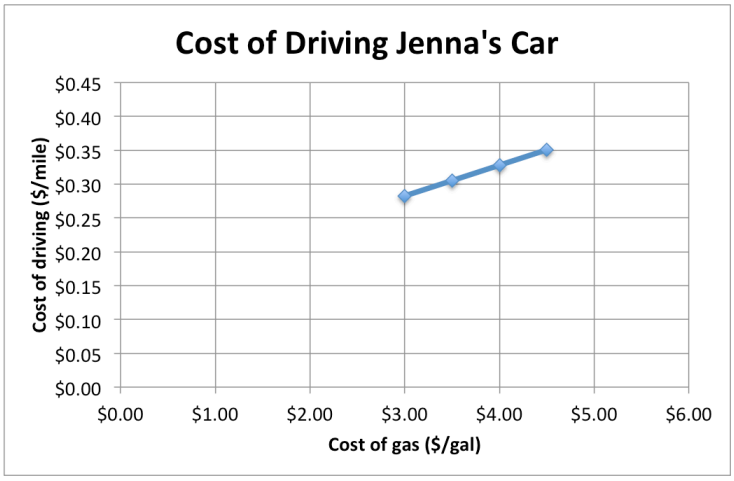
Table

Another way that you could have represented the relationship between the price of gas and the cost of driving the car is in a table that shows values of g and C as ordered pairs. An ordered pair is two values that are matched together in a given relationship. Tables are helpful for recognizing patterns and general relationships or for giving information about specific values. A table should always have labels for each column. The labels should include units when appropriate.

Price of Gas (dollars/gal)	Cost of Driving the Car (dollars/mile)
3.00	0.28
3.50	0.31
4.00	0.33
4.50	0.35

Graph

A graph provides a visual representation of the situation. It helps you see how the variables are related to each other and make predictions about future values or values in between those in your table. The horizontal and vertical axis of the graph should be labeled, including units.



Summary

Throughout this course, you will learn that having the skill to move between different forms and tools is important in problem solving. Alternating among the four representations of mathematical relationships is another example of this. In some cases, you may struggle writing an equation, but find that starting with a table is helpful. You might want a graph for a visual representation, but also need to express a relationship in words. It is important for you to be able to translate one form into another and choose which form is most useful in a specific situation.