Resource **Dimensional Analysis**

Quantitative reasoning skill: Ratios and unit rates

Unit rates are ratios with a denominator of 1, although they are not always written as fractions. For example, 60 mph is the same as $\frac{60 \text{ miles}}{1 \text{ hour}}$.

The language "miles per hour" implies that the operation is miles divided by 1 hour.

As another example, in 2012, the federal minimum wage was $7.25/hour. This means that an employee earns $7.25 for 1 hour of work, or $\frac{\$7.25}{1 \text{ hour}}$.

A worker may also be paid a weekly salary. If an advertisement states that a job pays $320 for a 40-hour work week, then that position can be compared to the previous job by converting to a unit rate:

$\frac{\$320}{40 \text{ hours}} = \frac{\$8 \cdot 40}{1 \cdot 40 \text{ hours}} = \frac{\$8}{1 \text{ hour}}$ The second job pays better.

Another way to think about the calculation above is as a division problem: $320 \div 40 = 8$.

This can be helpful when the numerator and denominator do not have a common factor.

Quantitative reasoning skill: Conversion factors

In the above example, the fraction was simplified by dividing out the common factor of 40/40 (which is equivalent to 1). A fraction that is a ratio of quantities can be equivalent to 1 even when the numerator and denominator are not the same number. However, it is necessary that the numerator and denominator represent equivalent quantities. For example, the following fractions are all forms of one:

$$\frac{16 \text{ ounces}}{1 \text{ pound}} \qquad \frac{1 \text{ mile}}{5{,}280 \text{ feet}} \qquad \frac{60 \text{ minutes}}{1 \text{ hour}}$$

These types of ratios are sometimes called conversion factors because they can be used to covert between units.

The example below shows how to set up a multiplication problem with the rate and the conversion factor to convert miles per hour to miles per minute.

$$\frac{35 \text{ miles}}{1 \boxed{\text{hour}}} \cdot \frac{1 \boxed{\text{hour}}}{60 \text{ minutes}}$$

Notice that the conversion factor is written so that the units of hours are in the numerator. This is because you want the *hours* label to divide out in the same way that common factors divided out in the weekly salary problem above. This leaves the units of miles/minute as shown here:

$$\rightarrow \frac{35 \text{ miles}}{1 \text{ hour}} \times \frac{1 \text{ hour}}{60 \text{ minutes}} \rightarrow \frac{35 \text{ miles}}{60 \text{ minutes}} \rightarrow \frac{0.58 \text{ mile}}{1 \text{ minute}}$$

Before continuing, make sure you can answer the following question:

- How was the 0.58 calculated?

Quantitative reasoning skill: Dimensional analysis

Dimensional analysis, unit analysis, or unit conversion are all names for the process of using conversion factors to set up and solve certain types of problems. Many professionals—including pharmacists, dietitians, lab technicians, and nurses—use unit analysis. It is also useful for everyday conversions in cooking, finances, and currency exchanges. Many people can do simple conversions without dimensional analysis; however, they will likely make mistakes on more complex problems.

The advantage of using dimensional analysis is that it is a way to check your calculations. While it is always important that you develop your own methods to solve problems, this is a time when you are encouraged to learn and use a specific method. Once you have learned dimensional analysis, you can decide when to use it and when to use other methods.

Suppose you needed to convert inches into feet and then into miles. The computation below shows how dimensional analysis can help you organize your work.

Example 1:

Unit labels can divide out in the same way that common factors do.

$$\frac{1{,}000 \text{ people}}{1{,}500 \text{ feet}} \times \frac{5{,}280 \text{ feet}}{1 \text{ mile}} =$$

$$\frac{1{,}000 \text{ people}}{1{,}500 \text{ feet}} \times \frac{5{,}280 \text{ feet}}{1 \text{ mile}} =$$

Multiply numerators.
Multiply denominators.
Simplify.

$$\frac{5{,}280{,}000 \text{ people}}{1{,}500 \text{ mile}} = 3{,}520 \text{ people per mile}$$

Remember that you must set up your conversion factor (the multiplier) so that matching labels appear in one numerator and one denominator.

The Khan Academy website has free videos and practice problem sets that you can use for additional review:

- https://www.khanacademy.org/math/arithmetic/rates-and-ratios/unit_conversion/v/converting-units-of-length

Some more extensive examples are shown on the next page.

Example 2:

Here is an example converting 1 year into seconds:

$$\frac{365 \text{ days}}{1 \text{ year}} \times \frac{24 \text{ hours}}{1 \text{ day}} \times \frac{60 \text{ minutes}}{1 \text{ hour}} \times \frac{60 \text{ seconds}}{1 \text{ minute}} =$$

$$\frac{365}{1 \text{ year}} \times 24 \times 60 \times 60 \text{ seconds} = 31{,}536{,}000 \text{ seconds per year}$$

Example 3:

A nurse or a pharmacist might need to know how many tablespoons are in a 250-mL (milliliter) bottle of medication. In this case, we are converting metric units (milliliters) to U.S. units (tablespoons).

There are 250 mL in one bottle, and a quick internet search indicates 1 tablespoon is about 14.79 milliliters.

$$\frac{250 \text{ mL}}{1 \text{ bottle}} \times \frac{1 \text{ tablespoon}}{14.79 \text{ mL}} = \frac{250 \text{ tablespoons}}{14.79 \text{ bottles}} \approx 16.9 \text{ tablespoons per bottle}$$

Example 4:

A father found some instructions on the internet for building a treehouse. The instructions were in metric units but he only had a standard English ruler. The instructions said the boards for the framing should be 2.5 meters long. How many inches should the dad measure?

$$\frac{2.5 \text{ meters}}{1 \text{ board}} \times \frac{100 \text{ cm}}{1 \text{ meter}} \times \frac{1 \text{ inch}}{2.54 \text{ cm}} = \frac{250 \text{ inches}}{2.54 \text{ board}} \approx 98.4 \text{ inches per board}$$

Of course, if you can connect to the internet, you could just search for “free online conversion calculator”!