Practice Assignment: Connection between Binomial and Normal Distributions

Questions 1–4: An exit poll is taken of 3,000 randomly selected voters in a statewide election. Let X denote the number in the sample who voted in favor of a special proposition designed to lower property taxes and raise the sales tax. Suppose that in the actual population, exactly 50% voted in favor of it.

1) Find the mean and standard deviation of the probability distribution of X. Round your answer to the nearest tenth.

   Answer: $\mu = np = (3,000)(0.50) = 1,500$, so the distribution has a mean of about $1,500$ voters; $\sigma = \sqrt{np(1-p)} = \sqrt{(3,000)(0.50)(0.50)} \approx 27.3861 \approx 27.4$, so the standard deviation is about 27 voters

2) Using the normal distribution approximation, give an interval in which you would expect X almost certainly to fall, if truly $p = 0.50$.

   Answer: We expect 99.7% of the voters who voted in favor of a special proposition designed to lower property taxes and raise the sales tax to fall between $\mu \pm 3\sigma$. Since $\mu - 3\sigma = 1,500 - 3(27.4) \approx 1,418$ *and* $\mu + 3\sigma = 1,500 + 3(27.4) \approx 1,582$, we expect the number of voters who voted in favor of a special proposition to lower property taxes and raise the sales tax to fall between $1,418$ and $1,582$.

3) Would it be surprising for the number of voters in the exit poll who voted in favor of a special proposition designed to lower property taxes and raise the sales tax to be 2,000?

   Choose the best option that explains your answer.

   a) Yes, it would be surprising because 2,000 falls outside the range (1418, 1582).

   b) No, it would not be surprising because 2,000 falls within the range (1418, 1582).

   c) No, it would be not surprising because 2,000 falls outside the range (1418, 1582).

   d) Yes, it would be surprising because 2,000 falls within the range (1418, 1582).

   Answer: a

4) Now, suppose that the exit poll number revealed that 1,786 of voters voted in favor of a special proposition designed to lower property taxes and raise the sales tax. What does this suggest about the actual value of p?

   Answer: It suggests that p is actually greater than 0.50.

5) In testing the assumption that the probability that a baby is a boy is 0.512, a geneticist obtains a random sample of 1,000 births and finds that 502 of them are boys. Using the continuity of correction, which one of the following describes the area under the graph of a normal distribution corresponding to the probability of observing more than 502 boys in a sample of 1,000 births?

   a) $P(X < 502.5)$

   b) $P(X > 501.5)$

   c) $P(X > 502.5)$

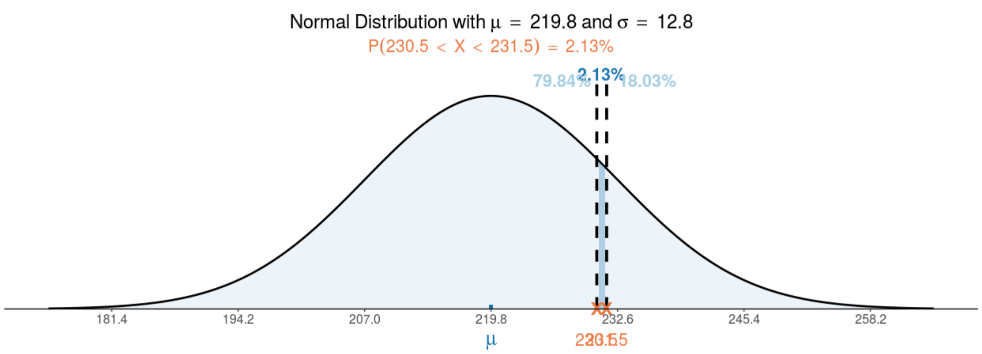
   d) $P(X \geq 503)$

   Answer: b

6) In professional tennis, 25% of challenges to a call by a referee are successfully upheld and the calls are overturned. Suppose that in a given season there will be 879 challenges made to referee calls. Let X be the number that will be successfully upheld, and assume that it is appropriate to use the normal distribution to approximate the probability of the distribution of X.

   Part A: If the 25% rate is correct, approximate the probability that among the 879 challenges, the number of overturned calls will be exactly 231 using the normal distribution. Round the values for the mean and the standard deviation to one decimal place when inputting them into the tool. Round the value of the probability to four decimal places.

   Answer: $P(X = 231) = P(230.5 < X < 231.5) = 0.0213$

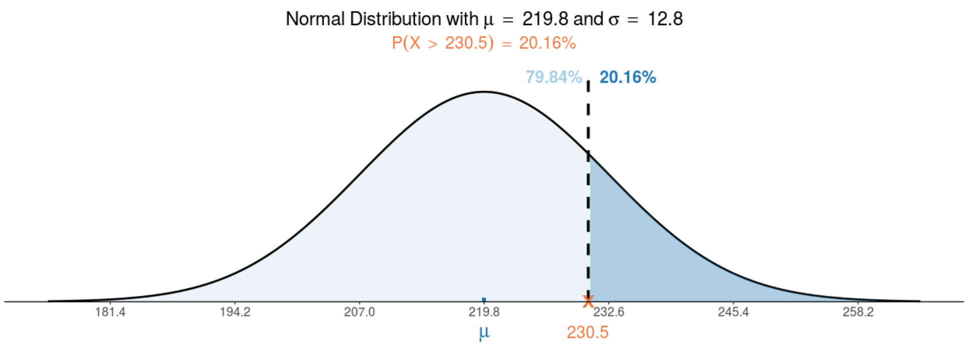


<u>**Normal Probability (Interval):**</u>

μ	σ	a	b	P(a < X < b)
219.8000	12.8000	230.5000	231.5000	2.125%

Part B: If the 25% rate is correct, approximate the probability that among the 879 challenges, the number of overturned calls will be 231 or more.

Answer: $P(X \geq 231) = P(X > 230.5) \approx 0.2016$



Normal Probability (Upper Tail):

μ	σ	x	P(X > x)
219.8000	12.8000	230.5000	20.16%

Part C: If the 25% rate is correct, is 231 overturned calls among 879 challenges a result that is unusual or significantly high?

Answer: The result of 231 overturned calls among 879 is not unusual or high compared to what we expected to see since $P(X \geq 231) = 0.2016$.

7) Suppose that the probability distribution of a male birth is 0.512. Would it be surprising to see exactly 21 boys in a random sample of 33 births?

To answer this question, use a normal distribution to approximate the binomial probability, and then find the probability using the binomial distribution. Compare the results. Are the approximations off by much?

Answers will vary.

Sample answer: In order to calculate the probability using the normal distribution, we must check the requirements to see if the normal distribution can be used to approximate the binomial probability. Since $np = (33)(0.512) = 16.896 \approx 17$ and $n(1-p) = (33)(0.488) = 16.104 \approx 16$, both the expected number of successes (baby being a boy) and the expected number of failures (baby being a girl) exceed 10, this binomial distribution can be approximated by a normal distribution.

Next, we find the μ and σ needed for the normal distribution: $\mu = np = 17$ and $\sigma = \sqrt{(33)(0.512)(0.488)} \approx 2.8714 \approx 2.9$, so $P(X = 21) = P(20.5 \leq X \leq 21.5) = 0.0504$. Using the binomial distribution, $P(X = 21) = 0.0508$. The probabilities are very close.