Practice Assignment: Test Statistics

1) As part of the in-class activity, you calculated a test statistic to decide whether a taste test conducted with students in Florida showed evidence of a preference for bottled water or tap water. Out of 22 students who participated, $\hat{p}$ = 20/22 = 0.909 preferred the taste of bottled water. The standard error for the sample proportion was 0.107 and the test statistic was 3.82.

Part A: Suppose you had treated tap water as a "success" and calculated $\hat{p}$ = 2/22 = 0.091. Use this sample proportion to calculate the test statistic.

Answer: –3.82

Part B: Compare the strength of evidence against the null hypothesis for the two test statistics. Select the best description.

a) There is more evidence against the null hypothesis when bottled water is treated as a "success" and $\hat{p}$ = 20/22 = 0.909.
b) There is more evidence against the null hypothesis when tap water is treated as a "success" and $\hat{p}$ = 2/22 = 0.091.
c) The evidence against the null hypothesis is the same, regardless of which type of water is treated as a "success."

Answer: c

2) When the sample size is large, a normal distribution can be used to model the values of the test statistic that would occur if the null hypothesis is true.

Part A: What is the mean for the null distribution of the test statistic?

Answer: 0

Part B: What is the standard deviation for the null distribution of the test statistic?

Answer: 1

Questions 3 and 4: In 2017, NFL overtime rules were called into question when the Atlanta Falcons lost the Super Bowl after losing the coin flip that determined who would receive the ball first in overtime. Under a perfectly balanced rule system, winning the coin toss would have no effect on the likelihood of winning the game. (Since winning the coin toss is random, the "better" team should win the coin toss half the time.) However, many believe that teams that win the coin flip at the start of overtime are more likely to win the game.[1]

[1] Sherman, R. (2017, February 6). *The NFL's overtime rules aren't fair—but neither are the alternatives*. The Ringer. https://www.theringer.com/2017/2/6/16042116/nfl-overtime-rules-super-bowl-li-patriots-falcons-62316a6f8e3c

3) Between 2012 and 2017, there were 82 NFL games that went into overtime and did not end in a tie. (Games before 2012 are not included here because there was a relevant rule change in 2012.) Of those, the teams that won the coin flip won 45 out of the 82 games.

Part A: State the null and alternative hypotheses in terms of the proportion of games won by the teams who won the coin flip.

Answer: $H_0: p = 0.5$, $H_A: p \neq 0.5$

Part B: Calculate the test statistic. (The standard error of the sample proportion is 0.055.)

Answer: z = 0.89

Part C: Interpret the value of the test statistic in context.

Answer: The sample proportion of games won by the teams that won the coin toss is 0.89 standard errors above what we would expect if the rule was fair.

Part D: Is it reasonable to use a normal distribution with a mean of 0 and a standard deviation of 1 to model the distribution of the test statistic that would occur if the null hypothesis is true?

Hint: Check the sample size condition.

Answer: Yes, because $np = 82(0.5) = 41$ and $n(1 - p) = 82(1 - 0.5) = 41$. Both of these values are greater than 10, so the sample size condition is met and the normal distribution can be used as a model.

Part E: Is the null hypothesis (the claim that the rule is fair) a plausible explanation for these data? Explain.

Answers will vary.

Sample answer: Yes, because test statistics have a normal distribution with a mean of 0 and a standard deviation of 1. The value z = 0.89 is not an unlikely value in this distribution. In other words, results like this would be likely to occur if the rule was fair.

4) Suppose the NFL had kept the same overtime rules for many seasons and the proportion of games won by the teams who won the coin flip remained very close to

the proportion observed between 2012 and 2017. How would the test statistic for this larger sample of games compare to the value calculated in Question 3, Part A?

a) The test statistic for the larger sample would be larger than the value calculated in Question 3, Part A.

b) The test statistic for the larger sample would be smaller than the value calculated in Question 3, Part A.

c) The test statistic for the larger sample would be approximately equal to the value calculated in Question 3, Part A.

Answer: a