Practice Assignment: Binomial Distributions

1) Match the phrase to the equivalent mathematical expression, where X counts the number of successes in a binomial experiment. Some mathematical expressions may be used more than once.

   Mathematical expressions:

$P(X = 3)$	$P(X > 3)$	$P(X \leq 3)$	$P(X < 3)$	$P(X \geq 3)$

   Part A: The probability of at least 3 successes

   Part B: The probability of exactly 3 successes

   Part C: The probability of fewer than 3 successes

   Part D: The probability that there are no more than 3 successes

   Part E: The probability of more than 3 successes

   Part F: The probability of having up to 3 successes

   Part G: The probability of 3 or more successes

Questions 2 and 3: We've seen binomial distributions used in a few contexts so far, but there are many more situations where the binomial model applies. One example involves testing sensitivity and specificity for medical tests.

- The **sensitivity** of a test is the true positive rate. In other words, the sensitivity is the probability that a person with the condition is correctly identified as having it.
- The **specificity** of a test is the true negative rate. In other words, the specificity is the probability that a person without the condition is correctly identified as not having it.

We'll consider these ideas in the context of rapid strep tests. Rapid strep tests are useful because the standard way to test for strep throat is to take a throat culture, which can take a few days to come back. If the rapid test comes back positive, the patient can get started on treatment right away and get better more quickly. If the test comes back negative but the physician is still concerned the patient might have strep, they will

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sometimes call for the throat culture at that point. On average, rapid strep tests have been found to have 86% sensitivity and 95% specificity.[1]

Let's suppose we are dealing with a rapid strep test with 86% sensitivity and 95% specificity.

2) 86% sensitivity means that of the people who actually have strep, 86% will test positive. This means that 14% of the people who actually have strep will test negative; these are called false negatives. A false negative can lead to a patient delaying the antibiotic treatment they need.

   Suppose we take a random sample of 30 people who have strep throat and are tested with this rapid strep test. Let X be the number of people who receive false negatives. Since we will not be sampling with replacement, assume that we are sampling from a population that is large enough so that the probability of success is close enough to the same in each trial that the sampling can be considered independent.

   Part A: Explain why this situation represents a binomial experiment, and give the values for n and p.

   Part B: What is the probability that at least 1 person receives a false negative? Use the *Binomial Distribution* tool at https://lumen-learning.shinyapps.io/binomialdist/ to determine this probability.

   Part C: What is the probability that fewer than 5 people receive false negatives?

   Part D: Would you be surprised if more than 3 people receive false negatives? Use statistical evidence to explain your answer.

3) 95% specificity means that of the people who do not have strep, 95% of them will accurately test negative. This means that 5% of the people who do not have strep will test positive for it; these are called false positives.

   Let's consider a random sample of 30 people who do not have strep and are tested with this rapid strep test. Let Y be the number of people who receive false positives. As in Question 2, assume we are sampling from a population that is large enough that we can consider the trials to be independent.

   Part A: What are n and p for this new binomial experiment?

[1] Cohen, J. F., Bertille, N., Cohen, R., & Chalumeau, M. (2016, July 4). *What is the performance of rapid tests for the diagnosis of strep throat in children?* Cochrane. https://www.cochrane.org/CD010502/ARI_what-performance-rapid-tests-diagnosis-strep-throat-children

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Part B: What is the probability that at most 2 people receive false positives?

Part C: What is the probability that none of the 30 people receive false positives?

Part D: Would you be surprised if 5 or more people receive false positives? Use statistical evidence to explain your answer.

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