

# Cheat Sheet:

# The Normal Distribution

## Essential Concepts

- A probability model includes all possible outcomes of a chance experiment and the probabilities associated with those outcomes. A probability model is also known as a probability distribution. For a probability distribution to be valid:
  - The outcomes are random events.
  - All outcomes are assigned a probability. The probabilities are numbers between 0 and 1.
  - The sum of all of the probabilities is 1.
- In a uniform probability model, each outcome has equal probability (for example, rolling a die where each face has a  $1/6$  probability of occurring)
- For a discrete random variable:
  - The values associated with the random variable of interest are numerical and discrete.
  - All possible values of the random variable are listed in a table or graph with each value having an associated probability greater than or equal to 0 and less than or equal to 1.
  - The sum of all probabilities in the table or graph equals 1.
- A continuous probability distribution is a probability distribution for a continuous random variable (an infinite and uncountable random variable). For all continuous random variables, the probability distribution can be approximated by a smooth curve called a probability density curve. The probabilities of a continuous probability distribution are represented as the area under a density curve.
- A normal distribution is a mathematical model with a smooth bell-shaped curve to describe the bell-shaped data distributions.

- A normal distribution has the following characteristics:
    - $X$  is a continuous random variable.
    - The mean is the center of the distribution which is symmetrical, the left side is a mirror image of the right side centered at the mean.
    - Bell shaped: There is one peak (unimodal) at the mean.
  - The standard deviation of a normal distribution,  $\sigma$ , will change depending on how spread out or flat the curve appears.
  - A normal distribution with a mean ( $\mu$ ) = 0 and a standard deviation ( $\sigma$ ) = 1 is called the standard normal distribution (or  $z$  distribution).
  - To compare  $x$ -values from different distributions, we can standardize the values into a standard normal distribution by converting the  $x$ -values into their respective  $z$ -scores.
  - When working with probability distributions, we often need to find the probability of values falling in certain regions of the distribution. There are three main types of probability calculations:
    - Upper tail  $P(X > a)$  finds the probability of the top percent, or events that exceed a given outcome.
    - Lower tail  $P(X < a)$  finds the probability of the bottom percent, or events below the given outcome. Lower tail is also used to find the percentile.
    - Interval finds the probability of an outcome occurring between two events. The probability of an outcome between two values can be found as  $P(a < X < b) = P(X < a) - P(X < b)$
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## Key Equations

### **z-score**

$$z = \frac{x - \mu}{\sigma}$$

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## Glossary

## **continuous probability distribution**

a probability distribution for a continuous random variable (an infinite and uncountable random variable)

## **discrete probability distribution**

a type of probability distribution that shows all possible values of a discrete random variable (countable or finite outcomes) along with the probabilities associated with those outcomes

## **interval probability**

the probability of observing a value between two specified values

## **lower tail probability**

the probability of observing a value less than a specified value; the area under a probability curve to the left of a specified point

## **normal distribution**

a mathematical model with a smooth bell-shaped curve to describe the bell-shaped data distributions

## **probability density curve**

the probability distribution that can be approximated by a smooth curve

## **probability model (probability distribution)**

all possible outcomes of a chance experiment and the probabilities associated with those outcomes

## **standard normal distribution**

a normal distribution with a mean  $\mu = 0$  and a standard deviation  $\sigma = 1$

## **uniform probability**

model a probability model where all possible outcomes have equal probability of occurring (e.g., rolling a fair die where each number has probability  $1/6$ )

## **upper tail probability**

the probability of observing a value greater than a specified value; the area under a probability curve to the right of a specified point