Cheat Sheet: Probability

Essential Concepts

- Probability is a measure that is associated with how certain we are of outcomes of a particular experiment or activity.
- A chance experiment involves making observations in situations where there is uncertainty about which of two or more possible outcomes will result.
- The sample space of a chance experiment is the collection of all possible outcomes for the experiment.
- For a chance experiment in which outcomes of the sample space are equally likely, the probability of an event is the number of outcomes in the event divided by the number of possible outcomes in the sample space.
- An outcome or collection of outcomes for a chance experiment is called an event.
- Theoretical probability is the probability that an event will happen based on pure mathematics. We consider the theoretical probability of an outcome to be the proportion of times the outcome would occur in the long run.
- For a chance experiment in which outcomes of the sample space are equally likely, the probability of an event is the number of outcomes in the event divided by the number of possible outcomes in the sample space.
- Theoretical probability is the probability that an event will happen based on pure mathematics, not by carrying out an experiment. The probability estimated from a chance experiment is called an empirical probability. The theoretical probability of an outcome is the proportion of times the outcome would occur in the long run.
- Facts about probabilities:
    - The probability of a certain event (an event that will happen) is equal to 1
    - The probability of an impossible event is equal to 0; this means that there are no possible outcomes for that event
    - Probabilities range from 0 to 1, including 0 and 1; So, for any event A, $0 \leq P(A) \leq 1$
    - Probabilities can be expressed as decimals, fractions, or percentages

- The Law of Large Numbers says that as we increase the number of times we repeat a chance experiment, the closer we can expect the empirical probability calculated from our chance experiment to be to the true probability.

- The complement of event A is denoted A' (read "A prime") or A^c (read "A complement). The complement of the event A consists of all outcomes that are NOT in A. It is calculated as $1 - P(A)$

- AND (∩) is the intersection of two events and means that both events must happen.

- An outcome is in the event A OR B, $A \cup B$, if the outcome is in A or is in B or is in both A and B.

- Probability Properties:

    - For any two events, A and B: $P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$
    - Two events are mutually exclusive if the probability that they both happen at the same time is zero. That is, if events A and B are mutually exclusive, then $P(A\ and\ B) = 0$
    - Therefore, for mutually exclusive events $P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B) = P(A) + P(B) - 0 = P(A) + P(B)$

- Sometimes, there are pairs of events for which one event has no effect on the probability of another event occurring. When this is the case, we say the events are independent. More formally:

    - Events A and B are independent if $P(A \cap B) = P(A) \cdot P(B)$

- A conditional probability, $P(A|B)$ pronounced A GIVEN B, is calculated based on the assumption that one event has already occurred. Conditional probabilities restrict the sample space.

- Two events are independent if $P(A|B) = P(A)$ and $P(B|A) = P(B)$

Key Equations

complement of an event

$P(A) + P(A') = 1$

or

$P(A') = 1 - P(A)$

independent events (multiplication rule)

$P(A) \times P(B) = P(A \cap B)$

Independent events (conditional probability)

$P(A \mid B) = P(A)$ and $P(B|A) = P(B)$

mutually exclusive events

$P(A \cup B) = P(A) + P(B)$

probability equation

$P(event) = \frac{number\ of\ outcomes\ in\ event}{number\ of\ all\ possible\ outcomes}$

probability property

for any two events, A and B, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Glossary

chance experiment

making observations in situations where there is uncertainty about which of two or more possible outcomes will result

complement

A' or A^c, consists of all outcomes that are not in A

conditional probability

calculated based on the assumption that one event has already occurred

empirical probability

probability estimated from a chance experiment

independent events

one event has no effect on the probability of another event occurring

Law of Large Numbers

as we increase the number of times we repeat a chance experiment, the closer we can expect the empirical probability calculated from our chance experiment to be to the true probability

mutually exclusive (disjoint)

when both event A and B do not occur or happen at the same time

$P(A\ given\ B) = P(A|B)$

the probability that event A will occur given that the event B has already occurred

$P(A\ and\ B) = P(A \cap B)$

the relative frequency of events A and B must happen in the same outcome

$P(A\ or\ B) = P(A \cup B)$

the relative frequency of either event A or B (or both) must happen in the outcome

$P(event)$

probability of an event

probability

a measure that is associated with how certain we are of outcomes of a particular experiment or activity

sample space

the collection of all possible outcomes for the experiment

theoretical probability

probability that an event will happen based on pure mathematics, not by carrying out an experiment

two-way (contingency) table

a table used in statistics to represent the relationship between two categorical variables