Cheat Sheet: Additional Concepts in Probability Distributions

Essential Concepts

- The probability distribution of a discrete random variable describes all possible values of the random variable, as well as the probability associated with each value.
    - The values associated with the random variable of interest are numerical and discrete.
    - All possible values of the random variable are listed in a table or graph, with each value having an associated probability greater than or equal to 0 and less than or equal to 1.
    - The sum of all probabilities in the table or graph equals 1.
- A Bernoulli trial is a chance experiment with the following three properties:
    1. There are exactly two possible outcomes of the chance experiment. We label one of them as a success and the other as a failure.
    2. The probability of success is the same for every trial. We call the probability of success p. Since the only two outcomes are success and failure, the probability of failure is the probability that the trial does not result in a success, so we can use the NOT probability rule to find that the probability of failure is $1-p$
    3. The trials are independent from one another.
- A binomial experiment is an experiment consisting of a fixed number, n, of independent Bernoulli trials that counts the number of successes out of n trials. Notice that the number of successes in a binomial experiment is a discrete random

variable. The distribution of this random variable is modeled with the binomial distribution.

- The formula for the probability of obtaining x successes from n independent trials where the probability of success is p is:

$$P(X=x) = (\text{number of ways to obtain } x \text{ successes in } n \text{ trials}) \cdot p^x \cdot (1-p)^{n-x}$$

- where p^x occurs because there are x successes, and $(1-p)^{n-x}$ occurs because if there are x successes and n trials total, there must be $n-x$ failures.
- Acceptance Sampling is a quality control technique used to assess the quality of a product or a batch of products. It involves inspecting a random sample from the batch and deciding whether to accept or reject the entire batch based on the quality of the sampled items. Acceptance sampling is commonly used in manufacturing, especially when testing the entire batch would be time-consuming or costly.
- For a binomial experiment with a probability of success p on n trials, the mean μ and standard deviation σ are defined as follows:
  - The mean of the number of successes is $\mu = np$.
  - The standard deviation of the number of successes is $\sigma = \sqrt{np(1-p)}$.
- Oftentimes, it is adequate to use the mean and standard deviation to describe the most likely values for the number of successes. For large n, the binomial distribution has an approximate bell shape. So, we can use the normal distribution to approximate the binomial distribution and conclude that nearly all possibilities for the number of successes fall between the mean and 3 standard deviations.
- The binomial distribution can be approximated well by the normal distribution when n is large enough so that the expected number of successes, np, and the expected number of failures, $n(1-p)$, are both at least 10. That is: The probability distribution will be approximately symmetric and bell shaped if

  $np \geq 10$ AND $n(1-p) \geq 10$

- In probability theory, a continuity correction is an adjustment that is made when a discrete distribution is approximated by a continuous distribution. We do this by adjusting the discrete whole numbers used in a binomial distribution so that any individual value, X, is represented in the normal distribution by the interval from $X - 0.5 \text{ to } X + 0.5$ or $X \# 0.5$.

Key Equations

conditions for normal distribution

$np \geq 10$ AND $n(1-p) \geq 10$

probability for independent trials

$P(A \cap B) = P(A) \cdot P(B)$

binomial distribution formula

$$P(X=x) = (\text{number of ways to obtain } x \text{ successes in } n \text{ trials}) \cdot p^x \cdot (1-p)^{n-x} = \frac{n!}{x!(n-x)!} \cdot p^x \cdot (1-p)^{n-x}$$

formula for the probability of obtaining `x` successes from `n` independent trials

$$P(X=x) = (\text{number of ways to obtain } x \text{ successes in } n \text{ trials}) \cdot p^x \cdot (1-p)^{n-x}$$

binomial mean

$\mu = np$

binomial standard deviation

$\sigma = \sqrt{np(1-p)}$

Glossary

acceptance sampling

a quality control technique used to assess the quality of a product or a batch of products

bernoulli trial

a chance experiment with two possible outcomes: success and failure

binomial experiment

an experiment consisting of a fixed number, n, of independent Bernoulli trials that counts the number of successes out of n trials

continuity correction

an adjustment that is made when a discrete distribution is approximated by a continuous distribution

discrete probability distribution

all possible values of a discrete random variable, as well as the probability associated with each value