Cheat Sheet: Additional Concepts in Probability

Essential Concepts

- A chance experiment involves making observations in situations where there is uncertainty about which of two or more possible outcomes will result.
- The probability of an event is a numeric measure of how likely the event is to happen.
- A probability is always a number from 0 to 1, inclusive (which means that 0 and 1 are included). Probability may be written as a percentage, from 0% to 100%, inclusive.
- A set is a collection of distinct objects, called elements of the set. The elements of a set can be either numbers or objects that have a particular characteristic in common. For example, we can define the set $A = \{...\}$ where the elements of the set are listed inside the curly bracket.
- A Venn diagram is a visual representation that illustrates the outcomes of a chance experiment. A Venn diagram represents each set by a circle/oval, usually drawn inside of a containing box representing the sample space. Overlapping areas indicate elements common to both sets.
- The complement of event A is denoted A' (read "A prime") or A^c (read "A complement"). The complement of an event A consists of all outcomes that are NOT in A.
- The intersection of two events, A and B, is denoted by $A \cap B$. A and B ($A \cap B$) means that the elements belong in both event A and also belong in event B. In term of probability, $P(A \text{ and } B) = P(A \cap B) =$ the relative frequency of event A and B with respect to the sample space.

- The union of two events, A or B, is denoted by $A \cup B$. The union of two events consists of the set of all elements in the collection of both events. The outcomes in the event A or B are the outcomes that are in event A, in event B, or in both event A and event B. In term of probability, $P(A \text{ or } B) = P(A \cup B) =$ the relative frequency of either event A or B (or both) with respect to the sample space.
- Mutually exclusive describes two or more events that cannot happen simultaneously. If event A and event B are mutually exclusive, then $A \cap B$ is an empty set and $P(A \cap B) = 0$.
- The conditional probability of A given B, denoted as $P(A \text{ given } B) = P(A|B)$, represents the probability of event A occurring given that event B has occurred.
- A tree diagram is a special type of graph used to determine the outcomes of an experiment. It consists of “branches” that are labeled with either frequencies or probabilities. In a tree diagram, probabilities are multiplied along the branches to find the probability at the end of a branch due to the Multiplication Rule of Probability.
- Bayes' Theorem is a fundamental concept in probability theory that allows us to update our beliefs about an event when new evidence or information becomes available.

Key Equations

bayes' theorem

$$P(A|B) = \frac{P(A) \times P(B|A)}{P(B)} = \frac{P(A) \times P(B|A)}{P(A) \times P(B|A) + P(A') \times P(B|A')}$$

or

$$P(A|B) = \frac{P(\text{the path of events A and B})}{\text{sum of } P(\text{paths containing event B})}.$$

probability of an event

$$P(\text{event}) = \frac{\text{number of outcomes in event}}{\text{number of all possible outcomes}}$$

conditional probability

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{P(A \cap B)}{P(B)}$$

complement of an event

$P(A) + P(\text{not } A) = 1$

or

$P(A) + P(A') = 1$

independent events

$$P(A \text{ and } B) = P(A) \times P(B)$$

mutually exclusive events

$$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B)$$

Glossary

bayes' theorem

a fundamental concept in probability theory that allows us to update our beliefs about an event when new evidence or information becomes available

chance experiment

making observations in situations where there is uncertainty about which of two or more possible outcomes will result

complement

A' or A^c, consists of all outcomes that are not in A

conditional probability

calculated based on the assumption that one event has already occurred

event

an outcome or collection of outcomes for a chance experiment

intersection

all elements in event A also belong in event B

mutually exclusive

two or more events that cannot happen simultaneously

$P(\text{event})$

probability of an event

probability

a numeric measure (the number of outcomes in the event divided by the number of possible outcomes in the sample space) of how likely the event is to happen

sample space

the list of all possible outcomes of a chance experiment

set

a collection of distinct objects, called elements of the set

tree diagram

a special type of graph used to determine the outcomes of an experiment

venn diagram

a diagram that represents each set by a circle/oval, usually drawn inside of a containing box representing the sample space

union

the set of all elements in the collection of both events